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New Chromatic and Symmetric Extensions of Dominance in Structured Graphs

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ABSTRACT: Domination theory is essential for understanding control and influence in network architectures. Traditional domination parameters are primarily concerned with coverage and adjacency, frequently overlooking the implications of colour limitations, internal connectedness and structural symmetry. In order to fill these gaps, this paper presents three new domination extensions: Symmetric Mirror Domination (SMD), Chromatic Restraint Domination (CRD) and Inter-Domination Sets (IDS). These extensions are examined on three structured graph families: Double Star Bridge Graph, Mirror Linked Path Graph and Skip Wheel Graph. These findings pave the way for further research into the structural and combinatorial aspects of graph domination.

1. INTRODUCTION

Graph theory is a powerful framework for describing relationships and control mechanisms in complex systems like communication networks, social systems and biological structures. In this field, domination theory investigates how a specific collection of vertices (dominators) can influence or monitor all others in a graph **[1]**. Several extensions to classical domination notions have evolved over time, including absolute domination, Roman domination, restricted domination, rainbow domination and distance k-domination each with extra limits or hierarchical elements to simulate real world complications. These variants have broadened the theoretical and practical scope of domination in fields including as fault-tolerant systems, sensor placement and resource distribution **[2]**.

In recent years, graph dominance theory has progressed beyond simple coverage models to more sophisticated frameworks that include colour, symmetry and connection. The Rainbow Domination (Bresar et al., 2021and Gao & Liu, 2023) emphasizes color-related domination by requiring vertices to be adjacent to dominators representing all colours in a k-labelling scheme **[3]**. This strategy effectively connects domination and graph colouring; however, it focuses on inter colour coverage **[8]**. Despite significant research, most extant domination models primarily address the coverage of non-dominating vertices, emphasizing external control rather than internal dynamics among dominators **[4]**. Furthermore, the relationship between domination and other graph features, such as symmetry and vertex colouring, has received less study **[6]**. This gap limits our capacity to simulate systems where dominance is both cooperative and constraint-sensitive, such as balanced communication structures or frequency-limited network.

Parallel to this, the importance of symmetry in domination has received more attention, particularly in research on mirror graphs, isomorphic substructures and twin networks (Imran et al.,2022., Malik & Han, 2024). These investigations address structural balance and redundancy, but they lack a formal framework that guarantees mirrored dominance **[5]**. Additionally, the significance of unity among dominators has been highlighted by advancements in connectivity-based domination, such as Connected, Secure and Paired domination (Alipour et al., 2023; Das & Pradhan, 2024) **[7]**.

To address these constraints, this study presents three new domination frameworks: Chromatic Restraint Domination (CRD), Symmetric Mirror Domination (SMD) and Inter-Domination Sets (IDS) each of which captures unique relationship characteristics among dominators. CRD introduces color-based constraints that prevent conflicts between adjacent dominators;

SMD models symmetrical or mirrored dominance within balanced network structures; and IDS investigates cooperative or overlapping interactions among multiple dominating sets within a shared graph environment. In general, this work broadens domination theory by incorporating chromatic, symmetric and inter dominance viewpoints, which improves our knowledge of control, interaction and balance in complicated graph frameworks.

2. PRELIMINARIES

Definition 2.1:

A set $D \subseteq V(G)$ is called an Inter-Dominating Set *(IDS)* if:

(i) every vertex $v \notin D$ is adjacent to atleast one vertex in D and

(ii) for every $u, v \in D$ there exists a vertex $x \in D$ such that both u and v are adjacent to x.

Example 2.1: For the Path Graph P_4

The vertices are $V_1 - V_2 - V_3 - V_4$

Let the domination set be $D = \{V_2, V_3\}$

Here, V_1 is adjacent to V_2 and V_4 is adjacent to V_3

Now, checking the interconnection for $u = V_2$ and $v = V_3$ where V_2 and V_3 are adjacent.

So, $x = V_2$ *or* $x = V_3$

Hence, $D = \{V_2, V_3\}$ is an IDS.

Therefore, the Inter domination number of the Path Graph P_4 is $\gamma_i(P_4) = 2$.

Definition 2.2:

For a proper colouring $f: V(G) \to \{1, 2, \ldots, K\}$, a set $D \subseteq V(G)$ is called chromatic restraint dominating set *(CRD)* if D dominates all vertices of G and for each colour i $D_i = D \cap f^{-1}(i)$ dominates all vertices of that colour.

Here, each colour forms a colour class, $f^{-1}(1) =$ *all vertices of colour 1.*

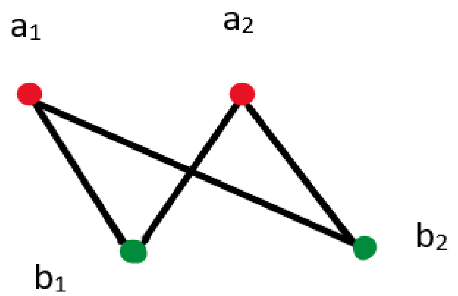
$f^{-1}(2) =$ *all vertices of colour 2.*

Example 2.2:

Consider the Bipartite graph of $K_{2,2}$ *as* shown in figure 2.2 (Left side: (a_1, a_2) (colour 1), Right side: (b_1, b_2) (colour 2))

Let the domination set be $D = \{a_1, b_1\}$ here a_2 is adjacent to b_1 and b_2 is adjacent to a_1.

Figure 2.2: A graph with 4 vertices.



The CRD condition:

Colour 1: $D_1 = \{ a_1 \}$ the colour 1 vertices (a_1, a_2) should dominate each other, but here a_1 dominates itself and a_2 is not adjacent to a_1. So, the condition fails.

Colour 2: $D_2 = \{ b_1 \}$ the colour 2 vertices (b_1, b_2) should dominate each other, but here b_1 dominates itself and b_2 is not adjacent to b_1. So, the condition fails.

Therefore, we take $D = \{ a_1, a_2, b_1, b_2 \}$ $\gamma_c(K_{2,2}) = 4,$ then each class dominates. Hence the chromatic restraint domination number of the Bipartite graph of $K_{2,2}$ is 4.

Definition 2.3:

A path graph $P_n = (v_1, v_2, \ldots, v_n)$ consists of n vertices arranged sequentially. Consider two isomorphic copies of path graph P_n denoted as $G_1 = P_n$ and $G_2 = P_n'$, where $V(G_1) = \{v_1, v_2, \ldots, v_n\}$ and $V(G_2) = \{v_1', v_2', \ldots, v_n'\}$.

For each $i \in \{1, 2, \ldots, n\}$, an edge $v_i v_i'$ is connected between the corresponding vertices G_1 and G_2. Then the resulting graph is $MLP_n = G_1 \cup G_2 \cup \{v_i v_i' \mid 1 \leq i \leq n\}$ is called the Mirror Linked Path Graph of order 2n.

Here, each vertex pair (v_i, v_i') is referred to as mirror pair and the edges $v_i v_i'$ as mirror edges. The graph MLP_n exhibits bilateral symmetry with respect to the mirror axis separating the two isomorphic path graphs G_1 *and* G_2.

Definition 2.4

A set $D \subseteq V(G)$ is a Symmetric Mirror dominating set if:

(i) D is a dominating set (every vertex not in D is adjacent to some vertex in D),

(ii) D satisfies mirror symmetry $v \in D \Rightarrow \phi(v) \in D$ *where* $\phi: V(G_1) \rightarrow V(G_2)$ is a mirror mapping between two isomorphic subgraphs G_1, G_2.

Example 2.3

Consider the graph has two paths (i.e., Top path (G_1): $V_1 - V_2$ and Bottom path

(G_2): $V_1' - V_2'$) as shown in figure 2.3.

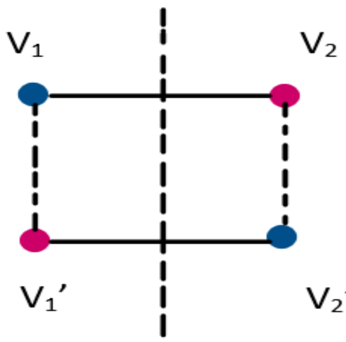


Figure 2.3: A graph with 4 vertices

From the figure 2.3, the vertical edges (V_1, V_1') and (V_2, V_2') are called mirror links. Let the mirror mapping be $\phi(V_1) = V_1'$ *and* $\phi(V_2) = V_2'$

Now checking the domination conditions, let $D = \{V_1, V_1'\}$, here V_2 is adjacent to V_1, V_2' is adjacent to V_1' and V_1, V_1' are already in D. Here, the condition satisfies.

The Mirror Symmetry be, $V_1 \in D \rightarrow \phi(V_1) = V_1' \in D$ and $V_1' \in D \rightarrow \phi(V_1') = V_1 \in D$

Therefore, the Symmetric Mirror Domination number of a Mirror Linked Path graph with four vertices (MLP_2) is 2.

Similarly, for the Mirror Linked Path graph with six vertices (MLP_3), the SMD is 2 and the Mirror Linked Path graph with eight vertices (MLP_4) the SMD is 4.

Definition 2.5:

A Skip Wheel Graph, denoted by $SW_{n,k}$ is a variation of the Standard wheel graph W_n which is constructed as:

- Start with a cycle graph $C_n = (v_1, v_2, \ldots, v_n)$ where $v_1, v_2, \ldots, v_n$ are called rim vertices.
- Then add a central vertex c, which is called as the hub.
- Next connect the hub c only to the alternate rim vertex (v_1, v_3, v_5).

Here n is the total number of vertices (including the hub), k is the skip parameter and v_i is the rim vertex.

Example 2.5

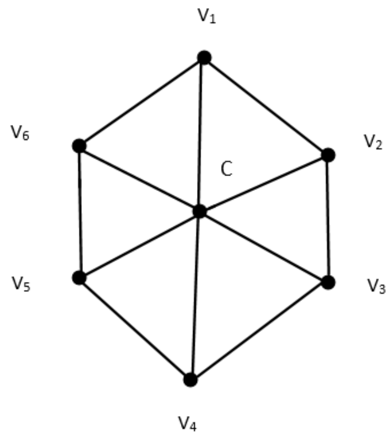


Figure 2.5: A Wheel Graph with 7 vertices

The vertex set of the Skip Wheel graph is written as $V(SW_{n,k}) = \{c, v_1, v_2, \ldots, v_n\}$ and the Edge set of Ship wheel graph is given as:

$$E(SW_{n,k}) = \{(v_i, v_{i+1}): 1 \leq i \leq n, v_{n+1} = v_1\} \cup \{(c, vi): i \text{ is odd})$$

Therefore, c is connected to v_1, v_3, v_5 to every second vertex of rim.

From the figure 2.5, Start with a cycle $C_6 = (v_1, v_2, v_3, v_4, v_5, v_6)$ and add a hub vertex c. Now, connect the hub to every alternate vertex, i.e., V_1, V_3 *and* V_5.

Let the vertex set be $V(SW_{6,2}) = \{c, v_1, v_2, v_3, v_4, v_5, v_6\}$. In Skip Wheel graph the edges are divided into Hub edges, Rim edges and Skip edges.

From the figure 2.5, the hub edges are $E_{hub} = \{cv_1, cv_3, cv_5\}$, hub c connects to alternate vertices. The rim edges are $E_{rim} = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1\}$. The skip edges are $E_{skip} = \{v_1v_3, v_2v_4, v_3v_5, v_4v_6, v_5v_1, v_6v_2\}$, these skip edges generate a second inner ring in which each rim vertex is connected to the vertex two positions distant in addition to its immediate neighbours.

$$|E(SW_6)| = |E_{rim}| + |E_{hub}| + |E_{skip}|$$
$$= 6 + 3 + 6$$
$$= 15$$

Therefore, the Skip Wheel graph $SW_{6,2}$ has 15 edges.

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Definition 2.6:

A Double Star Bridge graph $DSB_{m,\,n}$ is formed by two-star graphs $K_{1,\,m}$ and $K_{1,\,n}$. Let the centers be c_1 and c_2. Each center c_1, connects to m leaves that is $a_1, a_2\ldots, a_m$. Each center c_2 connects to n leaves $b_1, b_2, \ldots, b_n$. Here the two hubs are joined by a bridge edge c_1 and c_2.

The vertex set is defined as $V\,(DSB_{m,\,n}) = \{c_1, c_2, a_1, a_2, \ldots., a_m, b_1, b_2, \ldots, b_n\}$ and the edge set is defined as,

$$E\,(DSB_{m,\,n}) = \{c_1 a_i: 1 \leq i \leq m\} \cup \{c_2\, a_i: 1 \leq i \leq n\} \cup \{c_1, c_2\}$$

Since, it begins with two-star graphs, the total vertices are $m + n + 2$. To connect the two star graphs a new edge is added between the bridge c_1 and c_2.

Therefore, the total edges are $m + n + 1$.

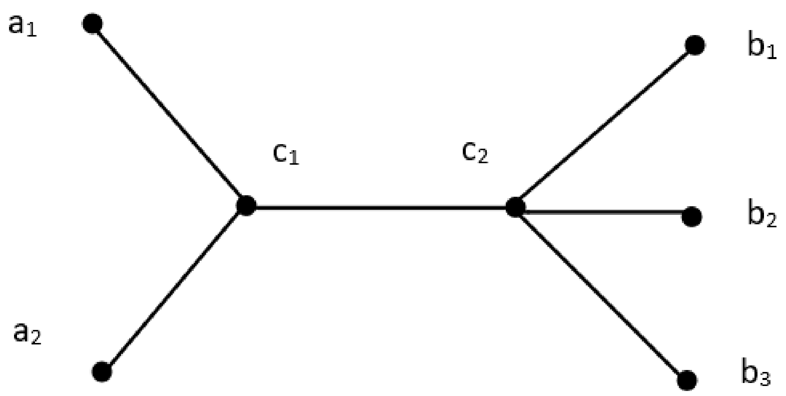

Figure 2.6: A star graph with 7 vertices

From the figure 2.6, if $m = 2$ *and* $n = 3$ then the $DSB\ (2, 3)$ has c_1, a_1, a_2 *from* S_2 *and* c_2, b_1, b_2, b_3 *from* S_3. From the above formula $m + n + 2$ the total vertices are $2 + 3 + 2 = 7$.

The edges of Double Star Bridge graph DSB (2, 3) is calculated as:

Edges in S_2 are $\{c_1\, a_1, c_1\, a_2\} \rightarrow\ 2$ *edges*

Edges in S_3 are $\{c_2\, b_1, c_2\, b_2, c_2\, b_2\} \rightarrow\ 3$ *edges*

Bridge edges are $\{c_1, c_2\} \longrightarrow 1$ *edge,*

Therefore, the total edges = $2 + \ 3 + 1 = 6$.

3. CHARACTERIZATION OF PROPOSED GRAPH MODELS

This study proposes three distinct domination models such as Chromatic Restraint Domination (CRD), Symmetric Mirror Domination (SMD) and Inter Dominating Sets (IDS) such as each of which is designed to represent a distinct structural feature of the graph families. These models expand on classical domination theory by incorporating new graph features such as colour constraints in CRD applied to Skip Wheel graph, structural symmetry in SMD which is examined on the mirror linked path graph and enhanced internal connectivity in IDS which is analysed within the double star bridge graph.

3.1 Symmetric Mirror Domination on Mirror Linked Path Graph (MLP_n)

Consider MPL_3, the mirror linked path with 3 vertices on each side as shown in figure 2.7.

Top path (G_1) is $v_1 - v_2 - \ v_3$ and Bottom path (G_2) is $v_1' - v_2' - v_3'$

The mirror links are $v_1 \leftrightarrow v_1',\ v_2 \leftrightarrow v_2',\ v_3 \leftrightarrow v_3'$

From the definition 2.4   A set $D \subseteq V(G)$ is a SMD if,

- D dominates the graph.
- $v \in D \Rightarrow \phi(v) \in D$

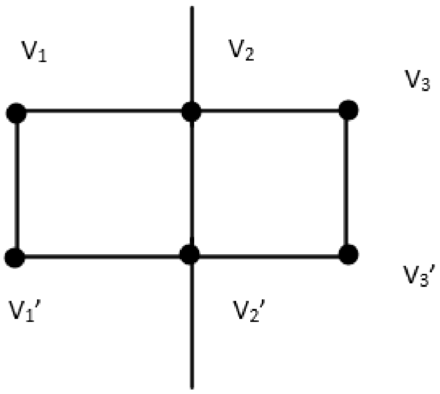

Figure 2.7: Mirror linked path graph with 6 vertices

From the figure 2.7, let us choose $D = \{v_2, v_2'\}$

Checking the domination and Symmetry, v_1 is dominated by v_2 and v_3 is dominated by v_2

v_1', v_3' is dominated by v_2'. All the vertices are dominated.

Here, $\phi(v_2) = v_2' \in D$ and $\phi(v_2') = v_2 \in D$, such that the mirror condition holds.

Hence, $D = \{v_2, v_2'\}$is a Symmetric Mirror Domination Set and $\gamma_s(MLP_3) = 2$.

The center pair v_2, v_2' controls both halves symmetrically, acting as a central dominator.

3.2 Chromatic Restraint Domination on Skip Wheel Graph (SW_n):

Consider a Skip Wheel graph SW_6 which functions similarly to a wheel graph W_6 but skips the alternate spokes.

The vertices are,

- center c
- The outer cycle is $v_1, v_2, v_3, v_4, v_5, v_6$
- The edges of outer cycle are $v_1 - v_2 - v_3 - v_4 - v_5 - v_6 - v_1$
- The center connected to v_1, v_3, v_5 (i.e. the alternate vertices).

Let us assign alternately around the cycle,

$f(v_1, v_3, v_5) = 1$

$f(v_2, v_4, v_6) = 2$

$f(c) = 3.$

From the definition 2.2, A set $D \subseteq V(G)$ is a CRD if, D dominates G and for each colour i, $D_i = D \cap f^{-1}(i)$ dominates all vertices of colour i.

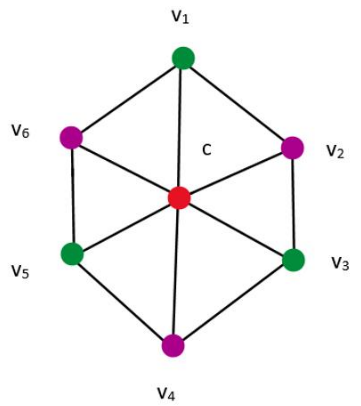

Figure 3.2: A Skip Wheel graph 7 vertices.

From the figure 3.2, choose $D = \{c, v_1, v_2\}$

Domination condition:

The center c dominates v_1, v_3 and v_5, were v_1 dominates v_2 and itself such as v_2 dominates v_1 and v_3.

Therefore, all the vertices are dominated.

Colour condition:

Colour 1: $D_1 = \{v_1\}$ dominates v_1, v_3, v_5

Colour 2: $D_2 = \{v_2\}$ dominates v_2, v_4, v_6

Colour 3: $D_3 = \{v_3\}$ dominates itself. Therefore, each colour class is self-dominating.

Hence, $D = \{c, v_1, v_2\}$ is a CRD set and the chromatic restraint domination number of a Skip Wheel graph with 6 vertices $\gamma_c(SW_6) = 3$.

As a result, each colour class is self-protected by its own-coloured vertices while the hub maintains overall control – a coloured balanced domination.

3.3 Inter-Dominating Set on Double Star Bridge Graph ($DSB_{m,n}$)

Two stars S_1, S_2 are connected by a bridge edge between centers c_1, c_2. From the definition 2.1, A set $D \subseteq V(G)$ is an Inter-Dominating Set if,

- Every vertex not in D is adjacent to some vertex in D.
- For every $u, v \in D$, there exists $x \in D$ such that both u, v are adjacent to x.

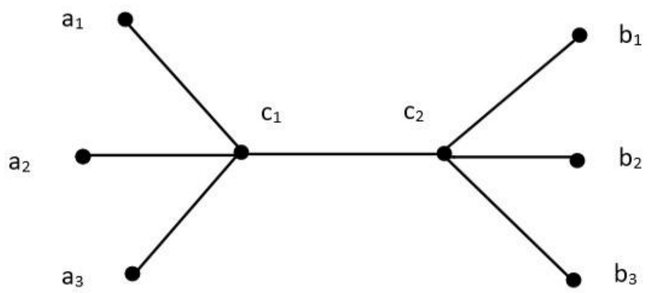

Figure 3.3: Double star bridge graph with 8 vertices

A Double Star Bridge graph *DSB (3, 3)*. From the figure 3.3, let us choose $D = \{c_1, c_2\}$. Here, *c_1 dominates a_1, a_2, a_3 and c_2 dominates b_1, b_2, b_3*. Therefore, all the vertices are dominated.

Inter- domination:

For $u = c_1$, $v = c_2$ here, $x = c_1$ or c_2 (because they are connected by a bridge). As a result, the Inter-Domination is satisfied.

Hence $D = \{c_1, c_2\}$ is an IDS set and $\gamma_i(DSB(3, 3)) = 2$, because each dominates its own side while remaining physically connected, the two-star centers create a mutually reinforcing pair that guarantees mutual impact among dominators.

Theorem 1:

Let MLP_n be a mirror linked network graph consisting of two isomorphic pathway P_n with corresponding vertices connected by mirror edges under a bijection $\phi: V(G_1) \rightarrow V(G_2)$ then,

$$\gamma_s(MLP_n) = 2\binom{n}{3}$$

Proof:

Let D be a symmetric mirror dominating set of MLP_n. *Let* $G_1 = (v_1, v_2, \ldots, v_n)$ and $G_2 = \{v_1', v_2', \ldots, v_n'\}$ be two paths forming the mirror copies of P_n in MLP_n.

By the definition of mirror domination, $v_i \in D$ if and only if $v_i' \in D$. Hence, each selected pair (v_i, v_i') acts as a symmetric dominator.

In P_n, a single vertex dominates almost three consecutive vertices, and the minimum number of dominators required to cover P_n is $\binom{n}{3}$. In MLP_n, each pair (v_i, v_i') dominates the corresponding triples (v_{i-1}, v_i, v_{i+1}) and $(v'_{i-1}, v_i', v'_{i+1})$ through adjacency along the paths and mirror edges. Therefore, the minimum number of symmetric dominating pairs required is $\binom{n}{3}$, and since each pair contributes two vertices, the total cardinality of D is $2\binom{n}{3}$.

There can be no smaller symmetric dominating set because reducing this would leave at least one mirror vertex undominated in both paths.

Hence, $\gamma_s(MLP_n) = 2\binom{n}{3}$.

Theorem 2:

Let SW_n be a skip wheel graph of order $n + 1$, where $n \geq 6$, consisting of a hub vertex connected to every alternate rim vertex of the cycle C_n. Then,

$$\gamma_c(SW_n) = 3.$$

Proof:

Let SW_n be a skip wheel graph formed from the cycle $C_n = (v_1, v_2, \ldots, v_n)$ with an additional hub vertex h. The hub h is adjacent to every alternate rim vertex, i.e. to the $\{v_1, v_3, v_5\}$. The remaining rim vertices $\{v_2, v_4, v_6\}$ are not connected directly to the hub.

Let $f: V(SWn) \rightarrow \{1, 2, 3\}$ be a proper 3-colouring of SW_n defined by:

$$f(h) = 1,\ f(v_i) = \begin{cases} 2, & \text{if } i \text{ is odd,} \\ 3, & \text{if } i \text{ is even.} \end{cases}$$

Thus, colour class $f^{-1}(1) = \{h\}$ corresponds to the hub, $f^{-1}(2)$ to the hub-connected rim vertices, and $f^{-1}(3)$ to the non-connected rim vertices.

Let $D = \{h, v_2, v_3\}$; D is a chromatic restraint dominating set (CRD), since h is adjacent to all colour 2-vertices (v_1, v_3, v_5, ….). Since, v_2 and v_3 dominate the remaining colour-3 and colour-2 on the rim.

Therefore, every vertex of SW_n is dominated by D.

Hence, the domination condition holds good.

Now, for each colour i we have

colour 1: $D_1 = D \cap f^{-1}(1) = \{h\}$ dominates itself.

colour 2: $D_2 = D \cap f^{-1}(2) = \{v_3\}$ which dominates all colour-2 vertices (v_1, v_5, v_7, ….).

colour 3: $D_3 = D \cap f^{-1}(3) = \{v_2\}$ which dominates all colour-3 vertices ($v_4, v_6, v_8, \ldots$).

Thus, every colour class is self-dominating, and satisfies the chromatic restraint condition. If any vertex of D is removed, at least one colour class will fail to be self-dominating.

Hence, $|D| = 3$ is minimal.

Therefore, the chromatic restraint domination number of the skip wheel graph satisfies $\gamma_c(SW_n) = 3$ *for all* $n \geq 6$.

Theorem 3:
For a double star bridge graph $DSB(m, n)$, obtained by connecting the central vertices of two-star graphs with a bridge edge. *Then* $\gamma_i(DSB(m, n)) = 2$, *for all* $m, n \geq 1$

Proof:

Let S_m and S_n be two-star graphs with centers c_1 and c_2, and leaves $\{u_1, u_2 \ldots, u_m\}$and $\{v_1, v_2, \ldots, v_n\}$ respectively.

Then the double star bridge graph $DSB(m, n)$ is formed by joining S_m and S_n through a brigde edge c_1c_2, by definition 2.6

Consider the set $D = \{c_1, c_2\}$. To show that D is an inter-dominating set of DSB (m, n).

Since c_1 is adjacent to all vertices of S_m and c_2 is adjacent to all vertices of S_n, every vertex of $DSB(m, n)$ is adjacent to at least one vertex in D.

Hence, D dominates the entire graph.

Also, for the inter condition, it shows that for any two vertices u, v ∈ D, there exists a vertex $x \in D$ such that both u and v are adjacent to x.

Here, the only pair is (c_1, c_2) and since $c_1c_2 \in E(DSB(m, n))$, both are adjacent to each other, satisfying the condition with $x = c_1$ (or equivalently $x = c_2$).

Therefore, $D = \{c_1, c_2\}$ is an Inter dominating set.

Also for any subset $D' = \{c_1\} \subset D$, no vertices in S_n are dominated.

Similarly, if $D' = \{c_2\}$, the vertices of S_m remain undominated.

Hence, $|D| = 2$ is the minimum possible size of an inter dominating set.

Thus, for all $m, n \geq 1$, $\gamma_i(DSB(m, n)) = 2$. Hence proves the theorem.

CONCLUSION:

This paper proposed and investigated three novel extensions of the classical domination concept, Chromatic Restraint Domination (CRD), Symmetric Mirror Domination (SMD) and Inter Dominating Set (IDS), each of which is intended to reflect a distinct structural aspect of graphs. These parameters were used on three different graph families: the Skip Wheel Graph, Mirror Linked Path Graph and the Double Star Bridge Graph. Further, how domination theory can be applied to include colouring constraints, structural symmetry and internal connectedness, broadening its theoretical applications are explained.

The study also demonstrates that these three new criteria maintain the fundamental logic of classical domination while applying practical and relevant constraints. The Chromatic Restraint Domination approach emphasizes chromatic self-sufficiency, which ensures that each colour class has adequate internal coverage. The Symmetric Mirror Domination model provides bidirectional control and redundancy in symmetric or mirrored networks, hence boosting structural stability. Meanwhile, the Inter-Dominating Set concept encourages connectivity among dominators, implying collaborative or cooperative control in complex systems. In both cases, the classical domination number serves as a bottom bound, indicating that these extensions are theoretically consistent while providing greater analytical insight.

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